Matrices

LEVEL – II

1. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A?

2. If A = $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ $is\ symmetric\ Find\ x.$

3. Find the product $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

4. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, find a + b + c .

5. If A = $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$, 2A + B + X = 0 , find X.

6. If 2 A + 3 B = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3 A + 2 B = $\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$, find A and B.

7. If A = $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, then verify that (AB)’ = B’ A’.

8. Find A such that A $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ = $\begin{bmatrix} 0 & -3 \\ 10 & 3 \end{bmatrix}$.

9. Show that A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies $A^2 - 4A - 5I = 0$, and hence find A^{-1}.

10. If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, Prove that A + A’ is a symmetric matrix.

11. Using elementary operation , find the inverse of the matrix A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

12. Find the matrix X, for which $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ X = $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$..

13. Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as a sum of symmetric and skew symmetric matrices.

14.If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$

15 . If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that f(A) = 0.

16. Solve for x and y: $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$.

17. If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find K so that $A^2 = 8A + K$

18. Express the following matrix as the sum of a symmetric and skew symmetric matrix and

Verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

19. Using elementary operations, find the inverse of the following matrix, :

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

20.Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of Rs. 11000 for the three values to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs. 2700, then.

i. Represent the above situation by a matrix equation and form Linear equations using matrix multiplication.

ii. Is it possible to solve the system of equations so obtained using matrices?

iii. Which value you prefer to be rewarded most and why?

LEVEL III

1.Find the inverse of the following matrix using elementary operations:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

2. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$

3. Find X, if $X + \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$.

4. Find x, if $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$.

5. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$,Prove that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric matrices

6. Using elementary operation, find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

7. If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ -1 & 4 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute A^2B^2

8. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that f(x) f(y) = f (x +y).

9. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$. find a,band k.

10.If AA' = I where A = $\begin{bmatrix} b & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ find a,b and c.

11. If A = $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16}.

12. If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and A^2 = $\begin{bmatrix} \propto & \beta \\ \beta & \alpha \end{bmatrix}$ Then prove that $\alpha + \beta = (a + b)^2$.

13. Prove that inverse of every square matrix if exist, is unique.

14. Show that diagonal element of a skew symmetric matrix is always zero.

15. If A = $\begin{bmatrix} cos\theta & i\ sin\theta \\ i\ sin\theta & cos\theta \end{bmatrix}$, then prove by principle of mathematical induction

that A^n = $\begin{bmatrix} cos\ n\theta & i\ sin\ n\theta \\ i\ sin\ n\theta & cos\ n\theta \end{bmatrix}$.

16. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, Show that $(aI + bA)^n = a^n I + n a^{n-1} bA$, where I is the

identity matrix of order 2 and n ∈ N.

Determinants

LEVEL II QUESTIONS:

SECTION A – (one mark)

1. Evaluate the determinant

$$\begin{vmatrix} 0 & sinx & -cosx \\ -sinx & 0 & siny \\ cosx & -siny & 0 \end{vmatrix}$$

2.If A is of order 3 and det A = 4, find det(3A).

3. Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

4. Show that the points A (a, b+c), B (b,c+a), C(c,a+b) are collinear.

5. Find values of k if area of triangle is 4 sq.units and vertices are (-2,0),(0,4),(0,k).

6. Find equation of line joining (1,2) and (3,6) using determinants.

7. If A is of order 3 and det (A) = -2, find det (AdjA)

8. Find k if $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & k \\ 1 & -2 & 1 \end{bmatrix}$ is singular.

9. If A is a square matrix of order 3 such that IAI = 4 what is I-5AI

SECTION B – (4 marks)

10. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$ = a^3

11. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = abc + bc + ca + ab =abc ($1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$)

12. Using properties of determinants prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

13. Using properties of determinants, Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & ab & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

14. Find minors and cofactors of the elements of the determinant

$$\begin{vmatrix} & & \\ & & \end{vmatrix}$$

$\begin{matrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{matrix}$ and verify that $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 0$.

15. Let A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1} A^{-1}$

16. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}

17. Examine the consistency of the system of equations,

$3x - y - 2z = 2$, $2y - z = -1$, $3x - 5y = 3$.

18.If a, b, c are in A.P. , find value of

$$\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$$

19. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$, $a \neq 0$

20. Using properties of properties of determinants, Prove that

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

SECTION C – (6 MARKS)

21. If A = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify that A (adjA) = | A|I , where I is the identity matrix. Also find A^{-1}

22. Solve the system of the following equations using matrix method.

$\frac{2}{x} + \frac{3}{Y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

LEVEL III QUESTIONS:

LEVEL III

(1 mark questions)

1. If A is a matrix of order 3 X 3 then verify that $\left(A^2\right)^{-1} = \left(A^{-1}\right)^2$

2. If A is a square matrix of order 3 such that $|adjA| = 64,\ find\,|A|$

3. If A $\begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, then find the value of k if $|2A| = k|A|$

4. Without expanding evaluate the determinant $\begin{vmatrix} \left(a^x + a^{-x}\right)^2 & \left(a^x - a^{-x}\right)^2 & 1 \\ \left(a^y + a^{-y}\right) & \left(a^y - a^{-y}\right)^2 & 1 \\ \left(a^z - a^{-z}\right)^2 & \left(a^z - a^{-z}\right)^2 & 1 \end{vmatrix}$,

where a > 0 and x,y are real numbers.

5. If A and B are non singular square matrices of the same order, then write the relationship between adj AB, adj A and adj B.

6. If A is invertible matrix of 3 X 3 and $|A| = 7$ then find $\left|A^{-1}\right|$

(4 mark questions)

1. If a, b and c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either a+b+c=0 or a=b=c.

2. Using properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

3.Using properties of the determinants , prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ac)^3$$

4.Using properties of determinants, prove that if x, y and z are different and

$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that 1+xyz=0.

5. Using properties of determinants, prove the following:

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\text{abc}(a+b+c)^3.$$

6.. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2a \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

7. Using properties of determinants, solve the following for x : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

8. Using properties of determinants, solve the following for x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

9. Using properties of determinants, show that ???? $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ = 4 abc

10.Using properties of determinants, prove the following: $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$

11. If *x*, *y*, *z* are the 10th, 13th and 15th terms of a G.P. find the value of $\begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}$

<u>(6 marks questions)</u>

1. If $= \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations x+2y+z=4 , −x+y+z=0 , x-3y+z=2.

2. If a, b and c are are positive and unequal , show that the following determinant is negative

$$\Delta= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

3. If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations:

$$2x + y + 3z = 3\ ,\ 4x - y = 3\ \ and - 7x + 2y + z = 2.$$

4. If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ $and\ B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. Use this to solve the following

system of equations. x - y =3 , 2x +3y + 4z =17 and y + 2z = 7 .

5. Examine the consistency of the following system of equations.

3x-y+7y=3, 2x+y+3z, x+4y-2z

6.Examine the consistency of the followi8ng system of equations

X –y + z=3, 2x+y-z=3, -x-2y+2z=1

7. Solve the following for CosA, CosB, CosC using Matrix method

a=b CosC+cCosB ,

b=cCosA+aCosC,

c=aCosB+bCosC